APPENDIX I

A MODEL OF THE HUMAN POSTURAL CONTROL SYSTEM

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Abstract

A nine degree of freedom pitch axis model of the human postural control system is proposed. Expressions for the kinetic and potential energies of the system are derived. The trunk, thigh, shank, foot, upper arm, forearm, and head are each modeled by a lumped mass at the element center of mass and an associated rotational inertia.

Background

Analysis related to an understanding of the human postural control system appears to be proceeding on five fronts. An excellent introduction to the first area, analytic representation of the force-velocity characteristics of human muscle tissue as an actuator, is provided in Milhorn's account [1] of the work on Houk [2]. This account is representative of a long preoccupation with muscle characteristics in reptiles, small vertebrates, and people dating back to 1935 [3]. Extensive references to more modern work are contained in Bahler [4], Vickers and Sheridan [5], Galiana [6], and Magdalena and McRuer [7]. The paper by Dewhurst [8] is also of interest in this area.

A second concentration of effort has occurred in the development of models for the physiological sensors involved in the postural control system; see, for example, the work on the vestibular system by Miery [9] and Young [10], and the work on the muscle spindle receptor by Houk [1,2] and by Gottlieb, Agarwal, and Stark [11], and Agarwal, et al. [20].

A third area, stimulated by a long-term Air Force interest in the capabilities of human pilots, is the view of certain elements of the postural control system seen in the context of human operator studies. An entree to this area of research is provided by the work of McRuer, Graham, Magdeleno, et al. [12-15], Shirley and Young [16], and Adams [17]; it is characteristic of this work that the human is seen from a black box viewpoint. The results and techniques developed in these studies are beginning to find application in analysis of the more prosaic automobile handling and control task; see Wier and McRuer [18] and Crossman [19].

A fourth area, quite distinct in viewpoint and methodology, is provided by the work of McGhee and Tomovic [25] in the theory of quadruped locomotion [21-24].

A representative mechanistic view of the postural control system for which there is some physiological support is provided by the block diagram developed by Houk [1,2] and redrawn here as Figure 1, which although developed with reference to the wrist-rotation reflex system, could in its essentials refer to the biceps-triceps pair or to any other distinct agonist-antagonist combination. The areas of research mentioned above concern themselves with different aspects of this block diagram. Results from the muscle force-velocity experimental studies have led to the form of the block diagram depicted in the pronator muscle and supinator muscle blocks; results from the sensor studies lead to the form given for the spindle receptor blocks, and human operator studies, together with some physiological support, lead to the form presented for the motor neuron complex, the spindle afferent nerves, and the alpha efferent nerves.

Such block diagrams usually have two general characteristics. First, the diagram "dead-ends" on the central nervous system block; apparently, this is where science ends and mysticism begins at our present state of knowledge. Secondly, this complex diagram relates to the control of a single joint angle -- a one degree of freedom system, hardly representative of the degree of muscular coordination commonly exhibited by Kittens and Kids.

The work of McGhee [21-25] and McRuer [12-15] attempts to meet these objections in two different ways.

In the majority of McGhee's work, the dynamic aspects of the system are suppressed, and concern is centered entirely on the question of what command inputs should be issued to the autopilot consisting of the proprioceptive loops if it is assumed that these loops function properly, or even better, ideally. Whether the leg motions that make walking possible are produced by the central nervous system in a purely voluntary mode of operation or whether a more autonomous type of reflex arc is involved is, of course, a question of long standing; McGhee provides convincing evidence for

the latter, and shows that useful quadruped gaits may be generated either by open loop (synchonous) or closed loop (asynchronous) digital networks. An open loop controller would consist of a clock-pulse driven shift register that issues leg commands at certain instants in time, independent of the actual state of the leg system at those times, whereas changes in the state of a closed loop controller would be triggered by satisfactory transfer of the leg system to the previously commanded state.

From the point of view of this paper, the importance of McGhee's work is that it begins to ask and provide technologically useful answers to questions about how the heretofore unassailable central nervous system might work, although it appears unlikely that we will soon have direct physiological evidence that this is the way it does work.

In McRuer's work, the known system dynamics are taken into account, and everything left over is lumped into a single black-box representation of the human operator performance, usually obtained in the form of a describing function for the box. A particularly good example of this approach is contained in Reference 14. Although this approach allows characterization of the human operator in a way that has extrapolative power to basically similar tasks, it has little synthetic power from the point of view of this paper -- it tells how the central nervous system works in certain tasks, but not why it works that way in circuitry terms. This difficulty is possibly due to the lack of any well-established describing function inversion techniques for describing functions that are both amplitude and frequency dependent [26].

Finally, the work of the General Electric corporation in the development of walking machines and remote manipulators should be mentioned [27,28]. Rather than to attempt the construction of a self-sufficient automaton, the GE approach is to imbed a human operator in a walking machine in such a way that through the use of force reflecting servos, the coordination capabilities of the operator are used to direct the machine's activities at a much higher force level. The concept is now being used in the development of off-road vehicles, machines for increasing the strength of an individual in lifting, and a variety of other applications. The basic difference between the GE research and that cited previously is that GE intends to utilize and extend the human muscular coordination capabilities, whereas the other approaches are directed towards an eventual replacement of the human, or toward purely scientific ends.

It is the purpose of this paper to pursue the behavior of the postural control system in a different direction than those outlined above. We would like to progress toward an understanding, from a systems point of view, of how some of the simpler common examples of large-scale human muscular coordination (for example, deep knee-bends) might be carried out, with the hope that someone might eventually develop an understanding of some of the more spectacular examples (like playing basketball).

To this end, it would be justifiable to gloss over some of the fine points other investigators have developed in the hope that taking such liberties will result in a systems model more appropriate to the large-scale phenomena we want to study; for example, only the gross features of muscle force-velocity characteristics will be retained on the grounds that from the point of view of understanding coordination, everything else known about muscle is irrelevant detail.

In more abstract language, we would like to develop an understanding of the goal-oriented integration of a number of proprioceptive loops into a more complete organism. We would accept simplified models of the individual loops to the extent necessary to achieve adequate complexity at the overall system level, with the goal of eventually achieving the development of a six degree-of-freedom simulation capable of predicting human capabilities and limitations in untested situations.

It is proposed to study this problem by means of simulation. A simplified, "pitch axis only" computer model of a human involving arms, legs, feet, head, and an (initially) rigid torso will be developed which can be dropped from various initial orientations, with the objective being to survive impact with a ground plane and return to an erect position. The work will follow the familiar pattern of simulation of a complex aerospace system: continuing development of the simulation will interact with continuing gathering of experimental data as critical questions are posed and understanding of the system progresses.

Our thesis is that the control algorithms necessary to provide command inputs to the proprioceptive loops would be more or less obvious from a consideration of the dynamic requirements of the specific task to be performed, depending on the complexity of the task. Obviously, an easy task should be selected first.

Kinetic and Potential Energies for the Model

In carrying out the derivation of the equations of motion of the postural control system model by use of Lagrange's equations, expressions for the potential and kinetic energies of the system are needed. A derivation of these expressions follows.

In terms of the generalized coordinates indicated in Figure 2, the total system kinetic energy may be expressed as the sum of the point mass kinetic energy and the kinetic energy due to the distributed nature of the arms, legs, etc.,

$$T = T_{DIST} + T_{POINT} \qquad (1)$$

The distributed kinetic energy is due only

to rotation of the various elements, and is easily obtained as

$$T_{DIST} = 1/2J_{TR}\dot{\theta}^2 + 1/2J_{HD}(\dot{\theta} + \dot{\zeta})^2 + 1/2J_{UA}(\dot{\theta} - \dot{\delta})^2 + 1/2J_{FA}(\dot{\theta} - \dot{\delta} - \dot{\varepsilon})^2 + 1/2J_{TH}(\dot{\theta} - \dot{\gamma})^2 + 1/2J_{SH}(\dot{\theta} - \dot{\gamma} + \dot{\beta})^2 + 1/2J_{FT}(\dot{\theta} - \dot{\gamma} + \dot{\beta} + \dot{\alpha})^2 \quad (2)$$

The kinetic energy due to the point masses is more difficult to obtain, as it involves repetitive application of the relative velocity theorem together with a long chain of coordinate resolutions to enable expression of the kinetic energy of each point mass in the form

$$T = 1/2M(V_a^2 + V_b^2)$$

where V_a and V_b are velocity components along any set of perpendicular axes, usually a set chosen to provide as much analytical simplicity as possible. A derivation of the V_a, V_b components for each of the masses follows. The absolute velocity of, say, the c.g. of the trunk is denoted by $\underset{\sim}{V}_{TR/O}$, while the relative velocity of, say, the hip with respect to the c.g. of the trunk is denoted by $\underset{\sim}{V}_{H/TR}$.

Unit vectors in a variety of directions are indicated on the diagram, and are denoted by, for example, $\underset{\sim}{u}_x$ and $\underset{\sim}{u}_y$. (See list of symbols).

The velocity components of each mass point must now be obtained along any convenient set of orthogonal axes, the choice of which varies during the analysis.

We have

$$\underset{\sim}{V}_{TR/O} = \dot{x}\underset{\sim}{u}_x + \dot{y}\underset{\sim}{u}_y \quad (3)$$

$$\underset{\sim}{V}_{H/TR} = \ell_{TRM}\dot{\theta}\underset{\sim}{u}_\theta \quad (4)$$

Using the relative velocity theorem, the absolute velocity of the hip can be expressed as the relative velocity of the hip with respect to the c.g. of the trunk plus the absolute velocity of the trunk:

$$\underset{\sim}{V}_{H/O} = \underset{\sim}{V}_{H/TR} + \underset{\sim}{V}_{TR/O} = \ell_{TRM}\dot{\theta}\underset{\sim}{u}_\theta + \dot{x}\underset{\sim}{u}_x + \dot{y}\underset{\sim}{u}_y \quad (5)$$

Resolving into $\underset{\sim}{u}_\theta$, $\underset{\sim}{u}_{TR}$ coordinates so that the vector addition can be performed,

$$\underset{\sim}{V}_{H/O} = \ell_{TRM}\dot{\theta}\underset{\sim}{u}_\theta + [-\dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_\theta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR} = [\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_\theta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR} = V_{H\theta}\underset{\sim}{u}_\theta + V_{HTR}\underset{\sim}{u}_{TR} \quad (6)$$

The absolute velocity of the CG of the thigh is then

$$\underset{\sim}{V}_{TH/O} = \underset{\sim}{V}_{TH/H} + \underset{\sim}{V}_{H/O} = \ell_{THM}\dot{\gamma}\underset{\sim}{u}_\gamma + \underset{\sim}{V}_{H/O} = \ell_{THM}\dot{\gamma}\underset{\sim}{u}_\gamma + V_{H\theta}\underset{\sim}{u}_\theta + V_{HTR}\underset{\sim}{u}_{TR} = \ell_{THM}\dot{\gamma}\underset{\sim}{u}_\gamma + [\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_\theta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR} \quad (7)$$

Resolving into $\underset{\sim}{u}_\gamma$, $\underset{\sim}{u}_{TH}$ coordinates,

$$\underset{\sim}{V}_{TH/O} = [\ell_{THM}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\underset{\sim}{u}_\gamma + [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\underset{\sim}{u}_{TH} \quad (8)$$

Similarly, the absolute velocity of the knee is given by

$$\underset{\sim}{V}_{K/O} = \underset{\sim}{V}_{K/H} + \underset{\sim}{V}_{H/O} = [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\underset{\sim}{u}_\gamma + [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\underset{\sim}{u}_{TH} \quad (9)$$

The absolute velocity of the center of the mass of the shank is given by

$$\underset{\sim}{V}_{SH/O} = \underset{\sim}{V}_{SH/K} + \underset{\sim}{V}_{K/O} = \ell_{SHM}\dot{\beta}\underset{\sim}{u}_\beta + \underset{\sim}{V}_{K/O} = \{\ell_{SHM}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)]\sin\beta\}\underset{\sim}{u}_\beta + \{[-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma}$$

$$- (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\underset{\sim}{u}_{SH} \quad (10)$$

The absolute velocity of the ankle is obtained by substituting ℓ_{SH} for ℓ_{SHM} in equation (10), giving

$$\begin{aligned}\underset{\sim}{v}_{A/O} &= \{\ell_{SH}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\beta]\}\underset{\sim}{u}_{\beta} + \{[-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\underset{\sim}{u}_{SH} \quad (11)\\ \underset{\sim}{v}_{FT/O} &= \underset{\sim}{v}_{FT/A} + \underset{\sim}{v}_{A/O} \\ &= \ell_{FTM}\dot{\alpha}\underset{\sim}{u}_{\alpha} + \underset{\sim}{v}_{A/O}\end{aligned}$$

Resolving equation (11) into $\underset{\sim}{u}_{\alpha}$, $\underset{\sim}{u}_{SH}$ coordinates, the absolute velocity of the c.g. of the foot is

$$\begin{aligned}\underset{\sim}{v}_{FT/O} &= [\ell_{FTM}\dot{\alpha} + \{[-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + y\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\cos\alpha + \{\ell_{SH}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\beta\}\sin\alpha]\underset{\sim}{u}_{\alpha} + [-\{\ell_{SH}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\beta\}\cos\alpha + [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta}$$

$$- \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\sin\alpha]\underset{\sim}{u}_{SH} \quad (12)$$

Expressions for the absolute velocities of the shoulder, upperarm, and forearm are obtained in similar fashion:

$$\begin{aligned}\underset{\sim}{v}_{S/O} &= \underset{\sim}{v}_{S/TR} + \underset{\sim}{v}_{TR/O} \\ &= -(\ell_{TR} - \ell_{TRM})\dot{\theta}\underset{\sim}{u}_{\theta} + \dot{x}\underset{\sim}{u}_{x} + \dot{y}\underset{\sim}{u}_{y} \quad (13)\end{aligned}$$

Resolving into $\underset{\sim}{u}_{\theta}$, $\underset{\sim}{u}_{TR}$ coordinates, the absolute velocity of the shoulder is obtained as

$$\begin{aligned}\underset{\sim}{v}_{S/O} &= -(\ell_{TR} - \ell_{TRM})\dot{\theta}\underset{\sim}{u}_{\theta} + [-\dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_{\theta} + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR} \\ &= [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_{\theta} + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR} \quad (14)\end{aligned}$$

The velocity of the c.g. of the upper arm is then given by equation (15):

$$\begin{aligned}\underset{\sim}{v}_{UA/O} &= \underset{\sim}{v}_{UA/S} + \underset{\sim}{v}_{S/O} \\ &= \ell_{UAM}\dot{\delta}\underset{\sim}{u}_{\delta} + \underset{\sim}{v}_{S/O} \\ &= \ell_{UAM}\dot{\delta}\underset{\sim}{u}_{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\underset{\sim}{u}_{\theta} + [\dot{x}\sin\theta + \dot{y}\cos\theta]\underset{\sim}{u}_{TR}\end{aligned} \quad (15)$$

Resolving into $\underset{\sim}{u}_{\delta}$, $\underset{\sim}{u}_{UA}$ coordinates gives

$$\underset{\sim}{v}_{UA/O} = \{\ell_{UAM}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta\}\underset{\sim}{u}_{\delta} + \{[(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta\}\underset{\sim}{u}_{UA} \quad (16)$$

The elbow velocity is obtained by replacing ℓ_{UAM} in equation (16) by ℓ_{UA}:

$$\begin{aligned}\underset{\sim}{v}_{E/O} &= \{\ell_{UA}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta\}\underset{\sim}{u}_{\delta} \\ &+ \{[(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta \\ &+ [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta\}\underset{\sim}{u}_{UA} \quad (17)\end{aligned}$$

The velocity of the forearm c.g. is obtained by the relative velocity theorem as

$$\chi_{FA/O} = \chi_{FA/E} + \chi_{E/O}$$

$$= \ell_{FAM}\dot{\varepsilon}\mu_{\varepsilon} + \chi_{E/O} \qquad (18)$$

Resolving into μ_{ε}, μ_{FA} coordinates yields

$$\chi_{FA/O} = \{\ell_{FAM}\dot{\varepsilon} + [\ell_{UA}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta]\cos\varepsilon - [(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta]\sin\varepsilon\}\mu_{\varepsilon} + [\ell_{UA}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta]\sin\varepsilon + [[(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta]\cos\varepsilon\}\mu_{FA} \qquad (19)$$

Finally, the absolute velocity of the head c.g. is given by

$$\chi_{HD/O} = \chi_{HD/S} + \chi_{S/O}$$

$$= \ell_{HDM}\dot{\zeta}\mu_{\delta} + \chi_{S/O} \qquad (20)$$

Resolving into μ_{δ}, μ_{HD} coordinates,

$$\chi_{HD/O} = \{\ell_{HDM}\dot{\zeta} - [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\zeta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\zeta\}\mu_{\zeta} + \{-[-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\zeta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\zeta\}\mu_{HD} \qquad (21)$$

Equations (3), (8), (10), (12), (16), (19), and (21) give the velocity components of the trunk, thigh, shank, foot, upper arm, forearm, and head respectively; these components are obtained in several different coordinate systems.

By combining equations (1) and (12), the kinetic energy of the trunk is

$$T_{TR} = 1/2J_{TR}\dot{\theta}^2 + 1/2M_{TR}\{\dot{x}^2 + \dot{y}^2\} \qquad (22)$$

From (1) and (8), the kinetic energy of the thigh is

$$T_{TH} = 1/2J_{TH}(\dot{\theta} - \dot{\gamma})^2 + 1/2M_{TH}\{[\ell_{THM}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]^2 + [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]^2\} \qquad (23)$$

From (1) and (10), the kinetic energy of the shank is

$$T_{SH} = 1/2J_{SH}(\dot{\theta} - \dot{\gamma} + \dot{\beta})^2 + 1/2M_{SH}[\{\ell_{SHM}\dot{\beta} + [\ell_{TH}\dot{\gamma} - \ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)]\sin\beta\}^2 + \{[-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}^2] \qquad (24)$$

From (1) and (12), the kinetic energy of the foot is

$$T_{FT} = 1/2J_{FT}(\dot{\theta} - \dot{\gamma} + \dot{\beta} + \dot{\alpha})^2 + 1/2M_{FT}\left\{\left[\ell_{FTM}\dot{\alpha} + \{[-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\gamma\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + \dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\cos\alpha + \{\ell_{SH}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\beta\}\sin\alpha\right]^2 + \left[-\{\ell_{SH}\dot{\beta} + [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma]\cos\beta - [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\sin\beta\}\cos\alpha + [-(\dot{x}\sin\theta + \dot{y}\cos\theta)\cos\gamma - (\ell_{TRM}$$

$$+ \dot{y}\sin\theta)\sin\gamma]\cos\beta - [\ell_{TH}\dot{\gamma} - (\ell_{TRM}\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta)\cos\gamma + (\dot{x}\sin\theta + \dot{y}\cos\theta)\sin\gamma]\sin\beta\}\sin\alpha\Big]^2\Big| \quad (25)$$

which clearly makes one wonder if it is worth the effort.

From (1) and (16), the kinetic energy of the upper arm is

$$T_{UA} = 1/2J_{UA}(\dot{\theta} - \dot{\delta})^2 + 1/2M_{UA}\Big[\{\ell_{UAM}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta\}^2 + \{[(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta\}^2\Big] \quad (26)$$

From (1) and (19), the kinetic energy of the forearm is

$$T_{FA} = 1/2J_{FA}(\dot{\theta} - \dot{\delta} - \dot{\varepsilon})^2 + 1/2M_{FA}\Big[\{\ell_{FAM}\dot{\varepsilon} + [\ell_{UA}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta]\cos\varepsilon - [(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta]\sin\varepsilon\}^2 + \{[\ell_{UA}\dot{\delta} + [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\delta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\delta]\sin\varepsilon + [[(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\delta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\delta]\cos\varepsilon\}^2\Big] \quad (27)$$

and, finally, the kinetic energy of the head is obtained from (1) and (21) as

$$T_{HD} = 1/2J_{HD}(\dot{\theta} + \dot{\zeta})^2 + 1/2M_{HD}\Big[\{\ell_{HDM}\dot{\delta} - [-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\cos\zeta - [\dot{x}\sin\theta + \dot{y}\cos\theta]\sin\zeta\}^2 + \{-[-(\ell_{TR} - \ell_{TRM})\dot{\theta} - \dot{x}\cos\theta + \dot{y}\sin\theta]\sin\zeta + [\dot{x}\sin\theta + \dot{y}\cos\theta]\cos\zeta\}^2\Big] \quad (28)$$

The total system kinetic energy is then given by

$$T = T_{TR} + T_{TH} + T_{SH} + T_{FT} + T_{UA} + T_{FA} + T_{HD} \quad (29)$$

The potential energy of the system is more easily obtained. By element, the potential energies of the trunk, thigh, shank, foot, upper arm, forearm, and head are given by (30)-(36),

$$V_{TR} = M_{TR}gy \quad (30)$$

$$V_{TH} = M_{TH}g[y - \ell_{TRM}\cos\theta - \ell_{THM}\cos(\gamma - \theta)] \quad (31)$$

$$V_{SH} = M_{SH}g[y - \ell_{TRM}\cos\theta - \ell_{TH}\cos(\gamma - \theta) - \ell_{SHM}\cos(\gamma - \theta - \beta)] \quad (32)$$

$$V_{FT} = M_{FT}g[y - \ell_{TRM}\cos\theta - \ell_{TH}\cos(\gamma - \theta) - \ell_{SH}\cos(\gamma - \theta - \beta) - \ell_{FTM}\sin(\theta - \gamma + \beta + \alpha)] \quad (33)$$

$$V_{UA} = M_{UA}g[y + (\ell_{TR} - \ell_{TRM})\cos\theta - \ell_{UAM}\cos(\theta - \delta)] \quad (34)$$

$$V_{FA} = M_{FA}g[y + (\ell_{TR} - \ell_{TRM})\cos\theta - \ell_{UA}\cos(\theta - \delta) - \ell_{FAM}\cos(\theta - \delta - \varepsilon)] \quad (35)$$

$$V_{HD} = M_{HD}g[y + (\ell_{TR} - \ell_{TRM})\cos\theta + \ell_{HDM}\cos(\theta + \zeta)] \quad (36)$$

and the total system potential energy is

$$V = V_{TR} + V_{TH} + V_{SH} + V_{FT} + V_{UA} + V_{FA} + V_{HD} \quad (37)$$

which can be grouped as

$$V = \{M_{TR} + M_{TH} + M_{SH} + M_{FT} + M_{UA} + M_{FA} + M_{HD}\}gy - \{[M_{TH} + M_{SH} + M_{FT}]\ell_{TRM} - [M_{UA} + M_{FA} + M_{HD}](\ell_{TR} - \ell_{TRM})\}g\cos\theta - \{M_{TH}\ell_{THM} + [M_{SH} + M_{FT}]\ell_{TH}\}g\cos(\gamma - \theta) - \{M_{SH}\ell_{SHM} + M_{FT}\ell_{SH}\}g\cos(\gamma - \theta - \beta) - \{M_{FT}\ell_{FTM}\}g\sin(\theta - \gamma + \beta + \alpha) - \{M_{UA}\ell_{UAM} + M_{FA}\ell_{UA}\}g\cos(\theta - \delta)$$

$$- \{M_{FA}\ell_{FAM}\}g \cos(\theta - \delta - \varepsilon)$$

$$+ \{M_{HD}\ell_{HDM}\}g \cos(\theta + \zeta) \qquad (38)$$

or

$$V = M_{gy} - V_1 \cos\theta - V_2 \cos(\gamma - \theta)$$

$$- V_3 \cos(\gamma - \theta + \beta) - V_4 \sin(\theta - \gamma + \beta$$

$$+ \alpha) - V_5 \cos(\theta - \delta) - V_6 \cos(\theta - \delta + \varepsilon)$$

$$+ V_7 \cos(\theta + \zeta) \qquad (39)$$

where M is the total mass of the stick man, g is the gravitational acceleration, and the V_i are constants obtained by comparison of equations (38) and (39).

Lagrange's equations, in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \qquad (40)$$

where the q_i are the system generalized coordinates and the Q_i are the generalized forces arising from muscular contraction, require partial differentiation of the system kinetic energy as given by equation (29) with respect to all the generalized coordinates and their derivatives and partial differentiation of the system potential energy as given by equation (39) with respect to the generalized coordinates. With no approximations, the resulting expressions are too lengthy for inclusion here; digital computer symbolic processing techniques to assist in carrying out the more laborious and error-prone calculations with greater reliability are being investigated.

Conclusion

Successful completion of the derivation would provide a pitch-axis model of the human postural control system, valid for large angular deflections of the joints, which could be simulated by standard analog or digital simulation techniques.

This simulation would include the dynamic force-velocity characteristics of the muscles in addition to the dynamics of the controlled object. It would then be possible to investigate the design of control laws [29] which, when incorporated into the simulation, could be tested for agreement with experimental data on human subjects performing the same tasks.

In this paper, the derivation of the postural control system equations of motion has been carried to the point where it clearly is advisable to utilize a digital computer to do the bookkeeping involved in the coordinate transformations and the subsequent partial differentiation required by Lagrange's equations. Certainly the possibility of ever extending the model to the full number of degrees of freedom of the human body would require such an organized "infallible" approach. Computer techniques for assisting in this problem are currently being investigated in terms of the present problem.

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Subscript Convention

TR	↔	TRunk, CG thereof
TH	↔	THigh, CG thereof
SH	↔	SHank, CG thereof
FT	↔	FooT, CG thereof
HD	↔	HeaD, CG thereof
UA	↔	Upper Arm, CG thereof
FA	↔	Fore Arm, CG thereof
H	↔	Hip
K	↔	Knee
A	↔	Ankle
T	↔	Toe
S	↔	Shoulder
E	↔	Elbow
N	↔	Neck

List of Symbols

J_{TR}	=	Moment of inertia of trunk, slug-ft^2
J_{HD}	=	Moment of inertia of head, slug-ft^2
J_{UA}	=	Moment of inertia of upper arm, slug-ft^2
J_{FA}	=	Moment of inertia of fore arm slug-ft^2
J_{TH}	=	Moment of inertia of thigh, slug-ft^2
J_{FT}	=	Moment of inertia of foot, slug-ft^2
M_{TR}	=	Mass of trunk, slugs
M_{HD}	=	Mass of head, slugs
M_{UA}	=	Mass of upper arm, slugs
M_{FA}	=	Mass of fore arm, slugs
M_{TH}	=	Mass of thigh, slugs
M_{FT}	=	Mass of foot, slugs

ℓ_{TR} = Length of trunk, ft

ℓ_{UA} = Length of upper arm, ft

ℓ_{TH} = Length of thigh, ft

ℓ_{SH} = Length of shank, ft

ℓ_{FT} = Length of foot, ft

ℓ_{TRM} = Distance from hip to trunk CG, ft

ℓ_{HDM} = Distance from shoulder to head CG, ft

ℓ_{UAM} = Distance from shoulder to upper arm CG, ft

ℓ_{FAM} = Distance from elbow to fore arm CG, ft

ℓ_{THM} = Distance from hip to thigh CG, ft

ℓ_{SHM} = Distance from knee to shank CG, ft

ℓ_{FTM} = Distance from ankle to foot, CG, ft

$\underset{\sim}{V}_{A/O}$ = Absolute velocity of ankle CG wrt origin, ft/sec

$\underset{\sim}{V}_{FT/O}$ = Absolute velocity of foot CG wrt origin, ft/sec

$\underset{\sim}{V}_{FT/A}$ = Relative velocity of foot CG wrt ankle, ft/sec

$\underset{\sim}{V}_{S/O}$ = Absolute velocity of shoulder wrt origin, ft/sec

$\underset{\sim}{V}_{S/TR}$ = Relative velocity of shoulder wrt trunk CG, ft/sec

$\underset{\sim}{V}_{UA/O}$ = Absolute velocity of upper arm CG wrt origin, ft/sec

$\underset{\sim}{V}_{UA/S}$ = Relative velocity of upper arm CG wrt shoulder, ft/sec

$\underset{\sim}{V}_{E/O}$ = Absolute velocity of elbow wrt shoulder, ft/sec

$\underset{\sim}{V}_{FA/O}$ = Absolute velocity of fore arm CG wrt origin, ft/sec

$\underset{\sim}{V}_{FA/E}$ = Relative velocity of fore arm CG wrt elbow, ft/sec

$\underset{\sim}{V}_{HD/O}$ = Absolute velocity of head CG wrt origin, ft/sec

$\underset{\sim}{V}_{HD/S}$ = Relative velocity of head CG wrt shoulder, ft/sec

$\underset{\sim}{V}_{TR/O}$ = Absolute velocity of trunk CG wrt origin, ft/sec

$\underset{\sim}{V}_{H/TR}$ = Relative velocity of hip wrt trunk CG, ft/sec

$\underset{\sim}{V}_{H/O}$ = Absolute velocity of hip wrt origin, ft/sec

$\underset{\sim}{V}_{TH/O}$ = Absolute velocity of thigh CG wrt origin, ft/sec

$\underset{\sim}{V}_{TH/H}$ = Relative velocity of thigh CG wrt hip, ft/sec

$\underset{\sim}{V}_{K/O}$ = Absolute velocity of knee wrt origin, ft/sec

$\underset{\sim}{V}_{K/H}$ = Relative velocity of knee wrt hip, ft/sec

$\underset{\sim}{V}_{SH/O}$ = Absolute velocity of shank CG wrt origin, ft/sec

$\underset{\sim}{V}_{SH/K}$ = Relative velocity of shank CG wrt knee, ft/sec

θ = Angle of trunk with respect to vehicle, radians, positive CW

δ = Angle of upper arm with respect to trunk, radians, positive CCW

ζ = Angle of head with respect to trunk, radians, positive CW

ε = Angle of fore arm with respect to upper arm, radians, positive CCW

γ = Angle of thigh with respect to trunk, radians, positive CCW

β = Angle of shank with respect to thigh, radians, positive CCW

α = Angle of foot with respect to perpendicular to shank, radians, positive CW

x = Horizontal displacement of trunk CG, ft

y = Verticle displacement of trunk CG, ft

$\underset{\sim}{u}_{x}$ = Unit vector in positive x direction

$\underset{\sim}{u}_{y}$ = Unit vector in positive y direction

$\underset{\sim}{u}_{TR}$ = Unit vector directed from hip to trunk CG

$\underset{\sim}{u}_{TH}$ = Unit vector directed from hip to thigh CG

$\underset{\sim}{u}_{SH}$ = Unit vector directed from knee to shank CG

$\underset{\sim}{u}_{FT}$ = Unit vector directed from ankle to foot CG

$\underset{\sim}{u}_{H}$ = Unit vector directed from shoulder to head CG

$\underset{\sim}{u}_{UA}$ = Unit vector directed from shoulder to upper arm CG

$\underset{\sim}{u}_{FA}$ = Unit vector directed from elbow to fore arm CG

$\underset{\sim}{u}_{\theta}$ = Unit vector in direction of increasing θ

$\underset{\sim}{u}_{\gamma}$ = Unit vector in direction of increasing γ

$\underset{\sim}{u}_{\beta}$ = Unit vector in direction of increasing β

$\underset{\sim}{u}_{\alpha}$ = Unit vector in direction of increasing α

μ_δ = Unit vector in direction of increasing δ

μ_ϵ = Unit vector in direction of increasing ϵ

μ_ζ = Unit vector in direction of increasing ζ

V = Potential energy, slug-ft

V_{TR} = Potential energy of trunk, slug-ft

V_{TH} = Potential energy of thigh, slug-ft

V_{SH} = Potential energy of shank, slug-ft

V_{FT} = Potential energy of foot, slug-ft

V_{UA} = Potential energy of upper arm, slug-ft

V_{FA} = Potential energy of fore arm, slug-ft

V_{HD} = Potential energy of head, slug-ft

$V_1, V_2 \ldots V_7$ = Constants in potential energy expression, slug-ft

T = Kinetic energy, slug-ft

T_{TR} = Kinetic energy of trunk, slug-ft

T_{TH} = Kinetic energy of thigh, slug-ft

T_{SH} = Kinetic energy of shank, slug-ft

T_{FT} = Kinetic energy of foot, slug-ft

T_{UA} = Kinetic energy of upper arm, slug-ft

T_{FA} = Kinetic energy of fore arm, slug-ft

T_{HD} = Kinetic energy of head, slug-ft

g = Gravitational acceleration, 32.18 ft/sec^2

M = Total system mass, slugs

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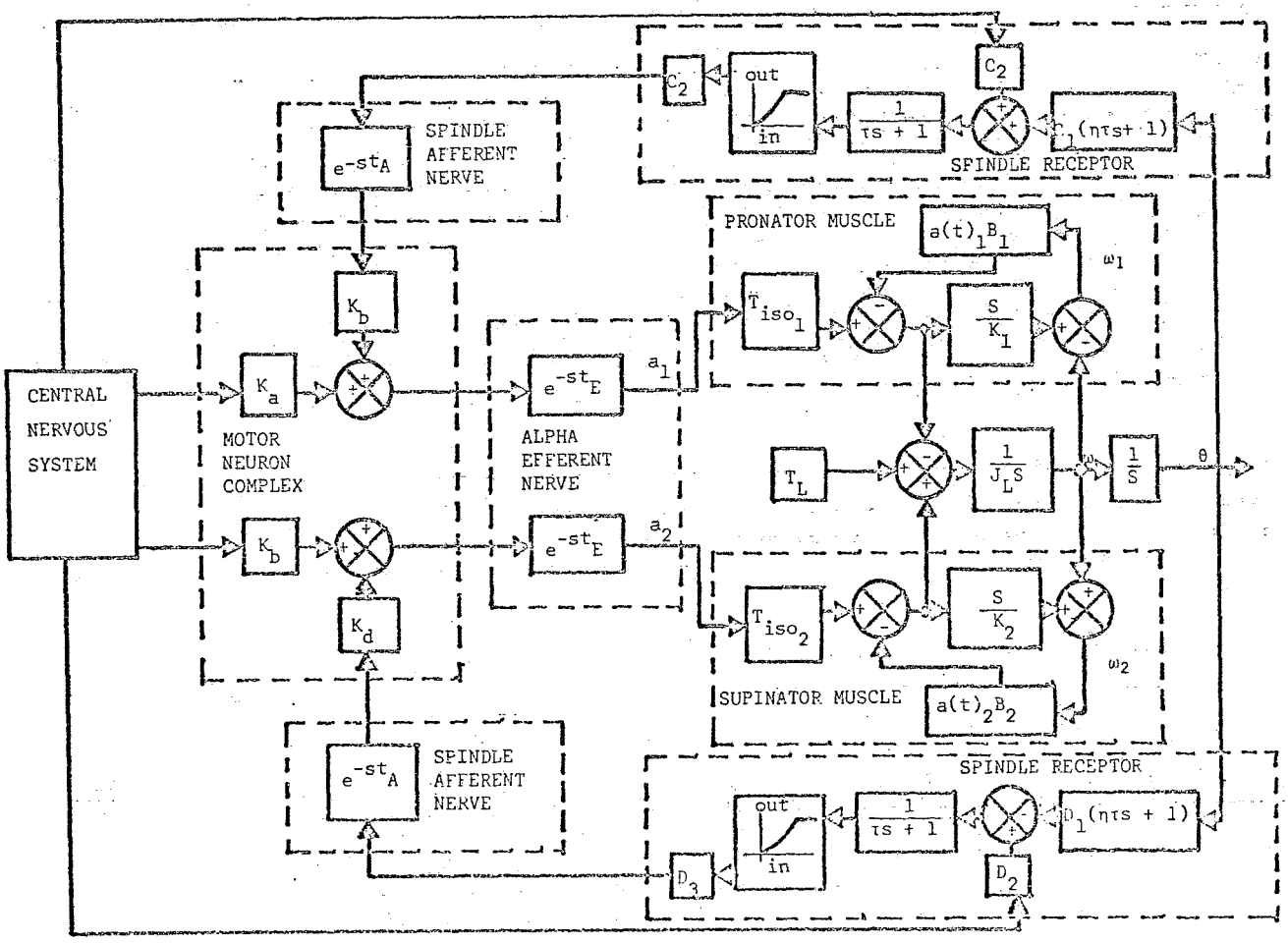

FIGURE 1. Postural Control System Block Diagram (From Milhorn, after Houk)

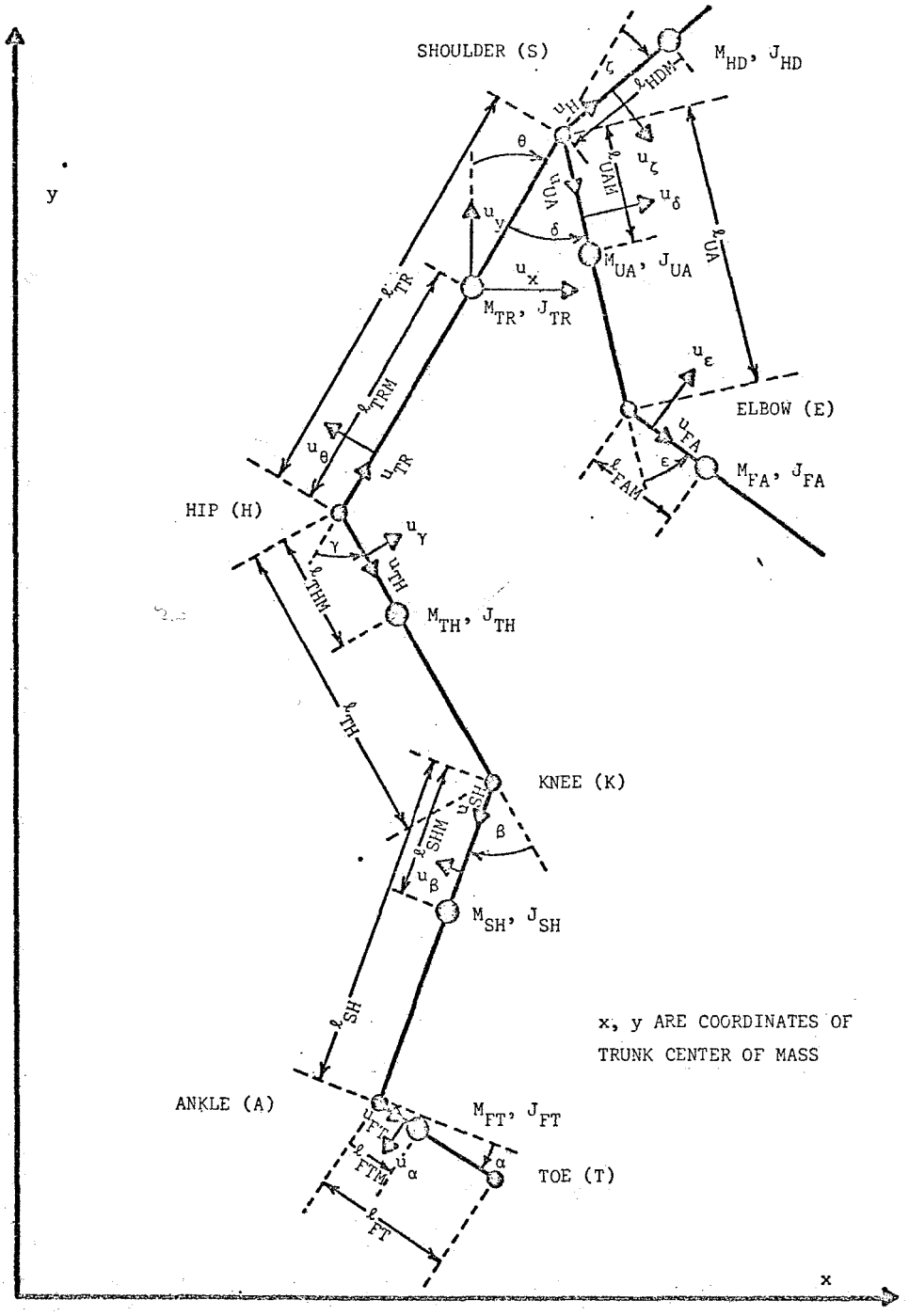

FIGURE 2. Coordinate System for Postural Control System Model